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Stress-Strain Relationship for Concrete in Compression Made of Local Materials

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ABSTRACT An attempt is made to evaluate the stress-strain relationship for concrete under uniaxial compression made of local materials.

Experimental program was carried out on concrete specimens to evaluate various parameters involved.

The proposed equations fit the experimental results with excellent agreement, hence, the distribution of stress, in concrete can be represented accordingly.

Introduction

For any sectional analysis, the evaluation and definition of the stress-strain relationship of concrete are required. Though a number of expressions are available, yet it may not be possible to define the relationship using one approach and completely represent the actual concrete behaviour in both the ascending and descending portion under the test conditions.

It is known to most investigators that this is due to the fact that stress-strain relationship is greatly influenced by a number of factors.

The shape of stress-strain curve is affected by the duration of loading and straining which is associated with the mechanism of internal progressive microcracking[1].

The theory of microcracking has led to only a better understanding of the behaviour of structural concrete section.

Investigators working on the ultimate moment capacity of R.C. section are quiet familiar with the available numerical approximation and empirical formulae to evaluate stress block parameters in which stress strain relationship comes first.

In this work, tests were carried out on concrete specimens using local materials brought from quarries that supply most of the southern region of Iraq with sand and gravel.

Simple equations in the form of a polynomial and a parabola are proposed based on the experimental results and suitable to local conditions of testing and designing. The results obtained are very encouraging.

Experimental Work

(a) Materials and Mix

In order to evaluate the stress-strain relationship experimentally, a series of tests were carried out on six cylinders (150 dia. and 300mm length) with an additional check on six prisms (150mm × 150mm × 400mm) which were casts of the same batch of concrete. Table 1 shows w/c ratio and mixes used to give different concrete strengths at the age of 28 days. No grading was carried out to the gravel or the sand, but minimum size of particle was kept to 20mm.

TABLE 1. Test results.

Concrete type	Mix	(W/C) ratio	Mean strength (MPa)
1	1: 1 : 2	0.40	43.5
2	1: 1 : 2	0.45	32.0
3	1: 2 : 4	0.55	27.7
4	1:2.5:3	0.60	25.3
5	1:2.5:3.5	0.66	16.7

Note: The mean strength is the average of six specimens.

Ordinary Iraqi cement was used. Casting, curing and testing were conducted according to Iraqi specifications.

(b) Strain Measurements

Strains were measured at the regions of uniform strain. Since concrete is a multiphase material, strain measurement is influenced by aggregate size. The recommendations given by Hanson and Kurvits[2] were followed by fixing a 60mm length electrical strain gauges (3 times the maximum aggregate size) on two adjacent sides. Demec points were also fixed on the two sides as an extra check on strain measurements.

Loading and strain measurement were carefully controlled. Continuous record of load and strain readings was obtained up to failure.

Maximum strength of concrete is expressed by the cylinder strength ($f_o = f'_c$).

Stress-Strain Relationship

It may be difficult to define the relationship by one approach due to the fact that the shape of uniaxial stress-strain curve of concrete is influenced by many factors.

Several hypotheses and approaches are available and fully employed by many investigators. In fact, some may differ in detail and others may differ significantly depending on how the factors affecting the relationship are evaluated and the manner in which testing conditions can be controlled[3-7].

However, there are several conditions that must be satisfied in any mathematical model, these are:

1. Point of origin, $f = 0$ at $\epsilon = 0$.

2. Slope of the stress-strain curve at the origin, $\frac{df}{d\epsilon} = E_c$ and $\epsilon = 0$.

3. Point of maximum stress $f = f_o$ at $\epsilon = \epsilon_o$, where $\frac{df}{d\epsilon} = 0$.

4. The analytical curve must satisfy the experimental data to show the ascending and descending portions.

In this regard, a carefully conducted set of experiments must be carried out. Besides, the model should be simple to use by designers.

With reference to Fig. 1, where the experimental results are shown and satisfying the above mentioned basic conditions, a single equation of a polynomial form for different types of concrete can be obtained as follows:

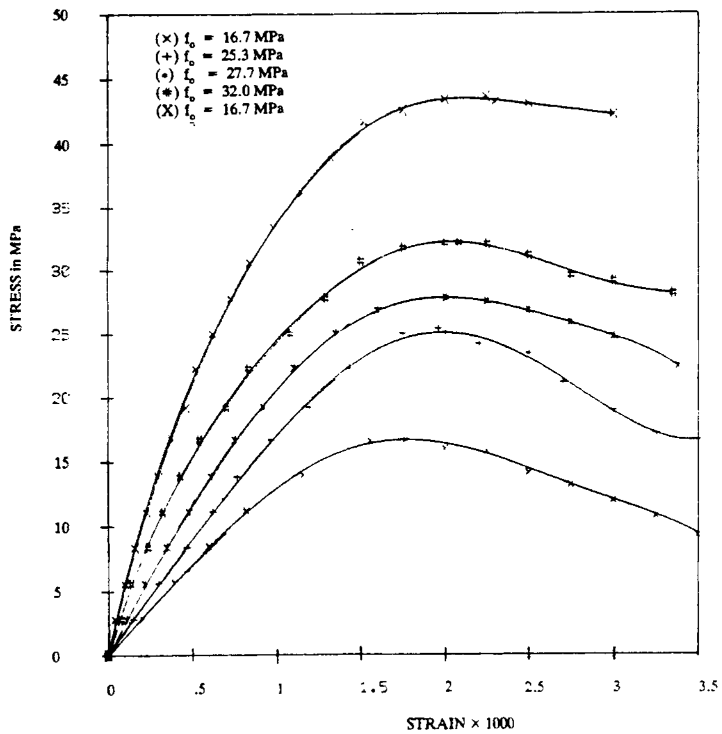

FIG. 1. Experimental results of stress-strain relationship for concrete.

$$\frac{f}{f_o} = A\left(\frac{\epsilon}{\epsilon_o}\right) + B\left(\frac{\epsilon}{\epsilon_o}\right)^2 + C\left(\frac{\epsilon}{\epsilon_o}\right)^3 \tag{1}$$

Coefficients A, B and C are evaluated by plotting the experimental results in non-dimensional form as shown in Fig. 2 and using the least squares polynomial curve fitting with, a third degree polynomial (n = 3) was selected in which equation (1) becomes:

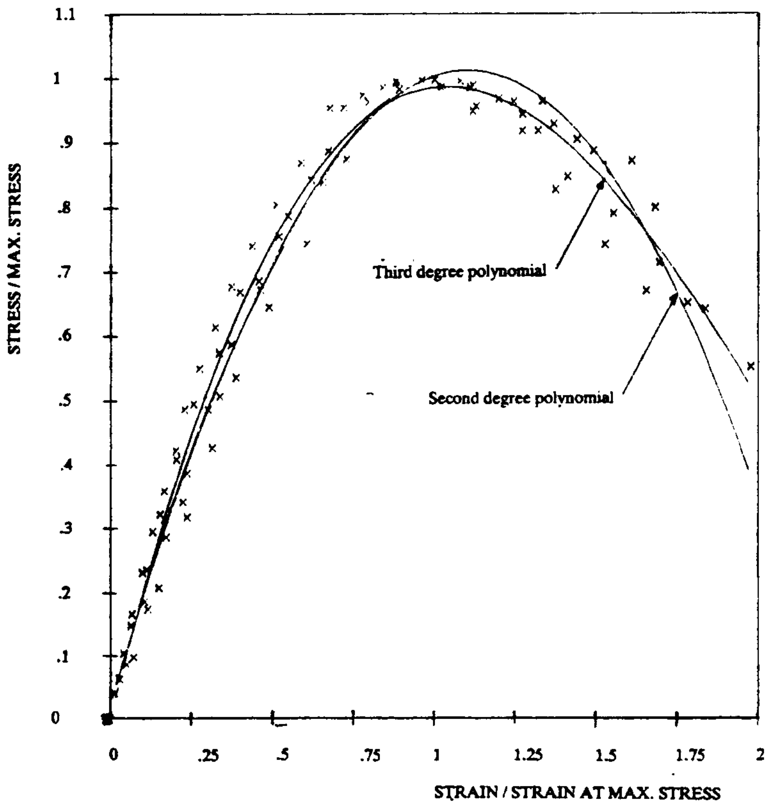

FIG. 2. Second and third degree polynomial fittings for experimental results.

$$\frac{f}{f_o} = 2.1\left(\frac{\epsilon}{\epsilon_o}\right) - 1.33\left(\frac{\epsilon}{\epsilon_o}\right)^2 + 0.2\left(\frac{\epsilon}{\epsilon_o}\right)^3 \tag{2}$$

Figure 2 shows a comparison between second and third degree polynomials. Obviously (n = 3) gives a better fitting. A fourth degree polynomial was also tried but was excluded from the analysis for its complexity.

Another simple form may be proposed for concrete stress-strain relationship similar to the form proposed by Desayi and Krishnan[8] and Carreira and Chu[9]:

$$\frac{f}{f_o} = \frac{R\left(\frac{\epsilon}{\epsilon_o}\right)}{1 + (R-1)\left(\frac{\epsilon}{\epsilon_o}\right)^{\beta}} \tag{3}$$

, in which, $\beta = \dfrac{R}{R-1}$

where R = material parameter depending on the shape of the stress-strain curve $= \dfrac{E_c}{E_o}$

E_c = modulus of elasticity of concrete,

E_o = max. stress f_o / strain at max. stress ϵ_o.

Various values of R were chosen as an attempt to find the value of (R) that has a good fit with the experimental data for different types of concrete selected from Table 2.

TABLE 2. Values of R based on the experimental results.

f_o (MPa)	E_c (GPa)	$\epsilon_o \times 10^{-3}$	E_o (GPa)	$R = \frac{E_c}{E_o}$
43.50	44.550	2.20 .	19.770	2.25
32.00	33.980	2.20	14.550	2.34
27.70	23.530	2.10	13.190	1.78
25.30	19.980	2.10	12.050	1.66
16.7	13.820	1.80	9.280	1.49

Average value of (R) = 1.90

Figure 3 shows a comparison between stress-strain relationship using equation 3 with different values of R. It is evident that equation 3 gives the best fitting when the value of R equals (1.9) which was found to be the average value of the set shown in Table 2. Therefore, equation 3 becomes:

$$\frac{f}{f_o} = \frac{1.9\left(\frac{\epsilon}{\epsilon_o}\right)}{1 + 0.9\left(\frac{\epsilon}{\epsilon_o}\right)^{2.1}} \tag{4}$$

This is also compared with equation 2 as shown in figures from 5 to 9 for different types of concrete used in this work. The comparison gives a good indication that the proposed equations 2 and 4 agree with the experimental results excellently.

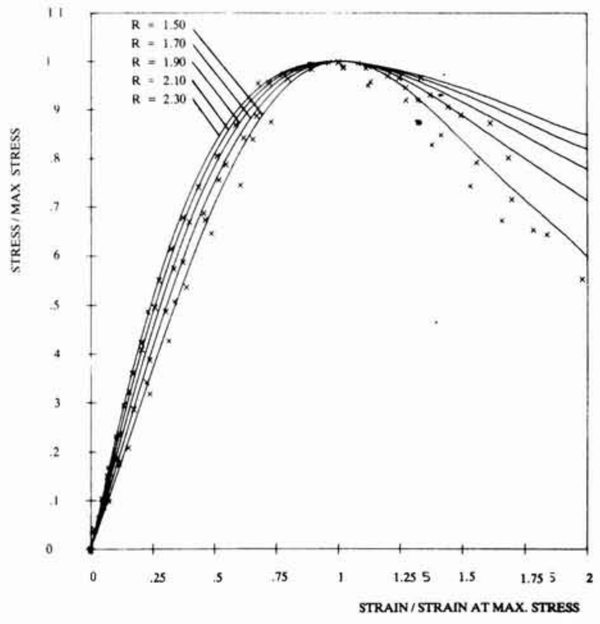

FIG. 3. Non-dimensional stress-strain relationship from Eq. (3) for various values of (R.

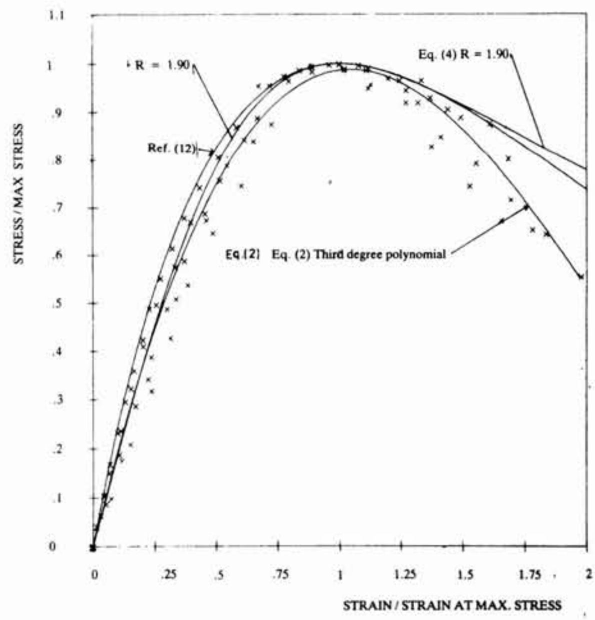

FIG. 4. Stress-strain relationships compared.

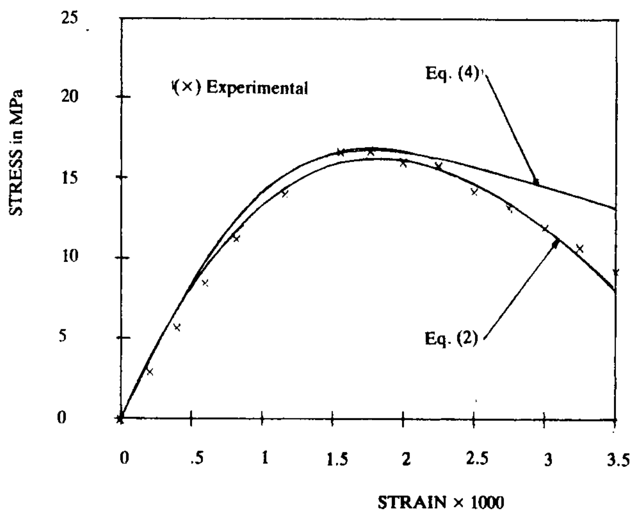

FIG. 5. Stress-strain relationships for concrete $f_o = 16.7$ MPa.

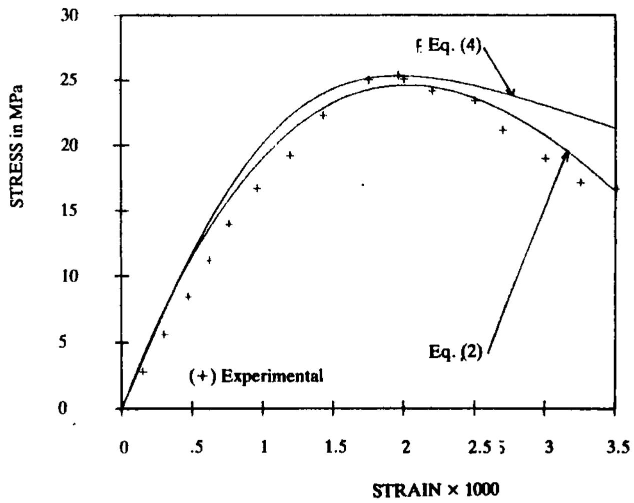

FIG. 6. Stress-strain relationships for concrete $f_o = 25.3$ MPa.

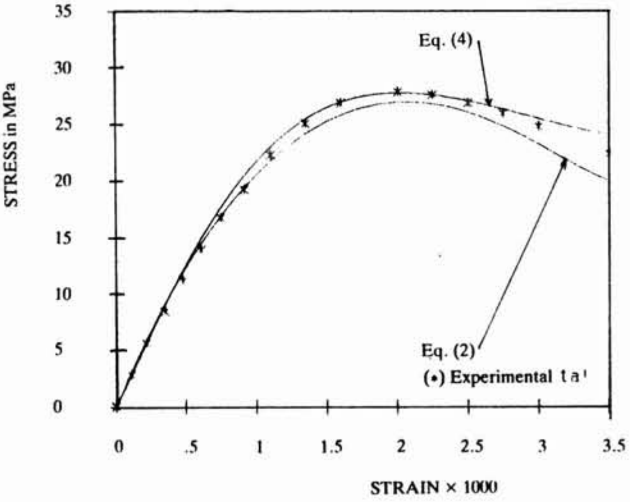

FIG. 7. Stress-strain relationships for concrete f_o = 27.7 MPa.

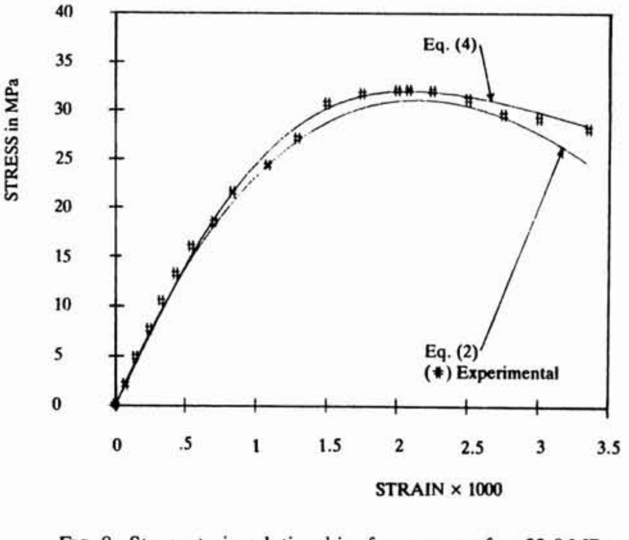

FIG. 8. Stress-strain relationships for concrete f_o = 32.0 MPa.

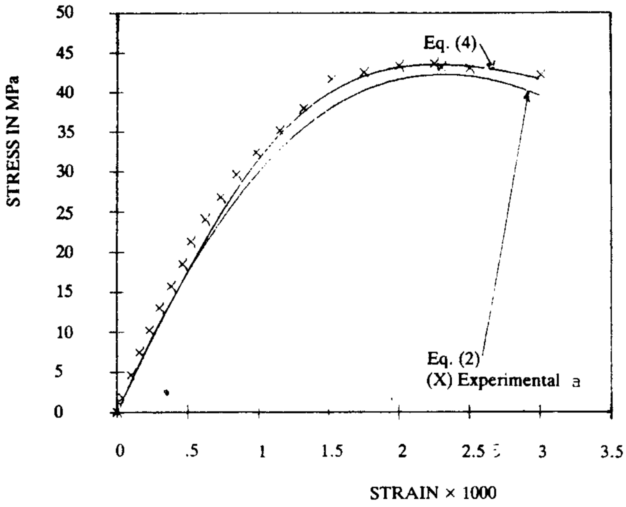

FIG. 9. Stress-strain relationships for concrete f_o = 43.5 MPa.

Estimation of ϵ_0

Values of strain corresponding to the peak point in the stress-strain curve is known as strain at maximum stress ϵ_0. The position of the peak point in the curve is influenced by compressive strength, rate of loading and straining, however, the peak point may be considered stable if the rate of straining and load duration are kept constant.

Popovics[10] reviewed various relationships of ϵ_0 in terms of f_0 which are commonly used.

It was found that the following proposed expression fits the experimental results[11]:

$$\epsilon_0 = 0.000875 \, (f_0)^{0.25} \tag{5}$$

, where $f_0 = f_c'$ in N/mm^2

Estimation of (ϵ_u)

ϵ_u is defined as the strain value at failure or the ultimate strain at failure. The difficulty involved in measuring this value experimentally has led some investigators to either assume certain values for ϵ_u or adopt the recommended value given by codes of practice. For this reason, a continuous record of stress and strain reading during the test beyond maximum stress, was recorded. Also, repeating the test on six specimens helped in giving reasonable values.

The following expression was found to represent the experimental data[11]:

$$\epsilon_u = \frac{0.0078}{(f_o)^{0.25}} \quad (6)$$

Conclusion

1. The work is based on testing concrete specimens made of local material to provide a relationship for stress-strain that designers can employ in the calculation of sectional moment capacity.

2. Equations 2 and 4 are proposed to define the stress-strain curve based on the experimental work conducted for this purpose. A remarkable agreement was obtained.

3. A comparison is made between the proposed equations and the work published by Smith and Young[12], in the form of exponential function which is shown in Fig. 4.

Figure 4 also shows that the proposed equation does represent the ascending and descending portions of the curve very well.

4. Maximum strain is defined as well as a fourth parameter the ultimate strain in the form of equations 5 and 6. A comparison between the experimental and the calculated values is made in Table 3. The comparison also shows that equations 5 and 6 do represent the experimental values well.

TABLE 3. Comparison between strain data.

f_o MPa	$\epsilon_o \times 10^{-3}$		$\epsilon_u \times 10^{-3}$		$\frac{\epsilon_o}{\epsilon_u}$ Exp.	$\frac{\epsilon_o \text{ Eq. (5)}}{\epsilon_u \text{ Eq. (6)}}$
	Exp.	Eq. (5)	Exp.	Eq. (6)		
43.50	2.20	2.247	3.00	3.037	0.73	0.74
32.00	2.20	2.081	3.35	3.280	0.65	0.63
27.70	2.10	2.007	3.38	3.400	0.62	0.59
25.30	2.10	1.962	3.51	3.470	0.59	0.57
16.70	1.80	1.768	3.60	3.850	0.50	0.46

$$\epsilon_o = 0.000875\,(f_o)^{0.25} \quad (5)$$

$$\epsilon_u = \frac{0.0078}{(f_o)^{0.25}} \quad (6)$$

5. There are now two equations for the stress-strain curve that can be employed easily by designers in the calculation of the ultimate sectional capacity with very well defined parameters based on the material that eventually be used in practice. In the author's opinion, the adopted relationships given by codes of practice, do not match the actual behaviour of concrete made of local materials as they are.

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علاقة الإجهاد - الانفعال للخرسانة تحت تأثير الضغط والمصنوعة من المواد المحلية

أنيس عبد الخضر محمد علي ، بلسم جاسم محمد فريد و عبد السلام إسماعيل محمد الجنابي
قسم الهندسة المدنية ، كلية الهندسة ،
جامعة البصرة ، العـراق

أجريت محاولة لإيجاد علاقة الإجهاد - الانفعال للخرسانة تحت تأثير الضغط والمصنوعة من المواد المحلية ، وكما هي وأي عدم إجراء فحص التدرج للحصى والرمل . شمل البرنامج العملي إجراء فحوصات مختبرية لمعرفة تصرُّف النماذج ولإيجاد المتغيرات ذات العلاقة .

كما أوضحت الدراسة أن المعادلة المقترحة لهذه العلاقة تعطي تطابقًا « جيداً » مع النتائج المختبرية والتصرف الفعلي للنماذج . ومن الممكن استعمالها في حساب توزيع إجهادات الضغط في مقاطع الأعضاء الخرسانية .