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Evaluation of the Efficiency of Shortened Low-Pass Filters Computed by Inverse Fourier Transform for Potential Fields of Spherical Bodies and Planar Regionals

AHMED A. AMMAR[1], KADRY M. FOUAD[2] and MAGDY L. MELEIK[2]
[1]Faculty of Earth Sciences, King Abdulaziz University, Jeddah, Saudi Arabia; and
[2]Nuclear Materials Corporation, Maadi-Kattamia Road, Cairo, Egypt.

ABSTRACT. The purpose of a separation filter may be defined as suppressing deep-rooted anomalies and faithfully reproducing shallow-rooted ones. Such separation could be performed by the transformation of gridded data using a set of coefficients computed by numerical evaluation of the Inverse Fourier Transform of the filter matrix.

A residual field due to an arbitrary spherical structure is chosen for testing and evaluating this method for obtaining a shortened matrix. The spherical model and planar regional are used merely due to the simplicity of calculations. The efficiency of the low pass filter is statistically computed and measured by the normalized residual root mean square (nrrms) between the theoretical and filtered fields.

The efficiency of filtering, due to different low pass filter parameters and matrix sizes, has been studied. Various inter-relationship curves, concerning the regionalization parameter (Q), matrix length (M), and depth to the causative body (Z), have been constructed for choosing a matrix which approximates the theoretical filter response best. It was found that the choice of a low pass filter (smoothing) parameter of $Q = 60$ and a matrix length of $M = 21$ is a compromise between the practical matrix size and the required efficiency of the low pass filter at different depth units (Z). It is also concluded that filter parameters and matrix lengths suitable for separation of deep spherical local bodies are efficiently used for shallower ones.

Introduction

The quantitative use of the potential field data is limited because of the low resolving power of the methods and because of the ambiguity in their interpretations. Despite these limitations, there is justification for getting as much out of the data as possible. Qualitative and quantitative interpretations could be made more objective by constructing the residual maps of the observed field. Residual maps have been used by geophysicists to bring into focus local features which tend to be obscured by the broader features of the regional field.

Several types of matrices have been devised by which uninteresting variations may be removed from a two-dimensional set of potential field data in order to emphasize those variations which may prove more interesting. The purpose of a separation filter may be defined as suppressing regional fields and faithfully reproducing sharp local anomalies. One well-known procedure consists of averaging the field values in the neighbourhood of a point and subtracting this average from the value at the point (*e.g.*, Peters 1949, Griffin 1949 and Henderson 1960).

It became convenient instead to treat the matrices as numerical filters applied to an equispaced grid of data in an x-y plane. Baranov (1975) performed the transformation by convolving the gridded data with a set of coefficients which are computed by numerical evaluation of the Inverse Fourier Transform of the frequency response of the filter matrix. He used the properties of the function with limited spectra as the basis of a method for filtering of potential fields. In all types of transformations, the problem has always been the design of matrices with optimum length (M), and low pass filter parameter (Q/q) at various depth units (Z). Large filters not only incur appreciable computational time but also results in the loss of valuable data points around the edges of the area under investigation.

The present work is concerned with the numerical evaluation of Baranov's technique (1975) and improvement of the conditions under which filtering proper is used, with the aim of optimizing the length and regionalization parameter (M and Q) of the low pass filter matrix for practical utilization at various depth units (Z).

Functions with Bounded Spectra in Two Dimensions

Filtering operation is based on the convolution of data with the mathematical function (Baranov 1975).

$$f(x) = F(x) * [\sin (\pi x/Q)/(\pi x)] \qquad (1)$$

which is a function with bounded spectra in one dimension having the effect of replacing the function F(x) with an arbitrary spectrum by a function f(x) whose spectrum is contained in the interval $-\pi/Q$ and $+\pi/Q$. The concept of filtering commonly applied to one dimension can be generalized to two-dimensional filtering by the function f(x,y) with bounded spectrum approximating the given function F(x,y) in at least-squares sense. If we consider

$$f(x,y) = \sum_k \sum_n X_{kn} \, E_k \, (x/Q) \, E_n \, (y/Q) \qquad (2)$$

where

$$X_{kn} = f(KQ, nQ) \text{ , and}$$

E_k & E_n are sampling functions.

The involved mathematical treatment (Baranov 1975) yields;

$$f(x,y) = (q/Q)^2 \sum_{(k)} \sum_{(n)} F(kq,nq)\ E_{k(q/Q)}\ (x/Q)\ E_{n(q/Q)}\ (y/Q)$$

where

$$E_{k(q/Q)}\ (x/Q) = [\sin\ (\pi/Q)\ (x - kq)] / [(\pi/Q)\ (x - kq)] \qquad (3)$$

F(kq,nq) is a given function sampled at an interval q approximated in the least squares sense by the function f(x,y), becoming smoother as the ratio Q/q increases. The latter ratio is considered as a smoothing or low pass filter parameter and at the same time is a regionalization parameter. If its value is large, equation (3) defines a regional anomaly.

Filter Coefficients

The numerical calculation of f(x,y) is implemented by the use of a matrix of coefficients given by;

$$C(k,n) = \{[\sin\pi k(q/Q)] / (k\pi)\}\ \{[\sin\pi n(q/Q)] / (n\pi)\} \qquad (4)$$

Equation (3) is used to calculate f(x,y) everywhere, by letting x = y = 0, then equation (3) becomes;

$$f(0,0) = \sum_{(k)} \sum_{(n)} F(kq,nq)\ [(\sin\pi kq/Q) / (k\pi)]\ [(\sin\pi nq/Q) / (n\pi)] \qquad (5)$$

From equations (4) and (5);

$$f(0,0) = \sum_{(k)} \sum_{(n)} C(k,n)\ F(kq,nq)$$

which shows that the value f(0,0) at the center results from the product of two matrices C and F. The matrix F represents the two-dimensional potential field data to be filtered, while the matrix C represents the calculated filter coefficients. The data are usually sampled at an interval taken as unit length, *i.e.,* q = 1.

Evaluation of The Technique

The satisfactory evaluation of this technique depends on the efficient separation of the regional component of the measured potential field. To investigate the accuracy of the technique, the method is applied to a theoretical field composed of a regional component imposed over a sharp local sphere anomaly. The regional field is expressed by;

$$0.3\ X + 0.2\ Y + 0.1\ XY$$

and the local sphere field is given by:

$$800/(X^2 + Y^2 + Z^2)^{1.5}$$

The filtered (separated) regional field is repeatedly computed using different assigned values of both the smoothing parameter (Q) and the matrix length (M) to help determine their optimal values at different depth units (Z). The object is to study how the agreement between a theoretical regional field and how the filtered regional version of the original regional field added to a sharp local sphere anomaly. This was effected by means of the calculation of the residual root mean square, rrms (Daniel *et al.* 1971) between the theoretical and the filtered fields, and then normalized, nrrms (Apell 1974) by relating this value to the maximum of the theoretical local anomaly at each depth.

Results and Discussion

Since anomalies consist of a wide band of space frequencies, the choice of the suitable matrix length (M) and the low pass filter parameter (Q) at various depth units (Z) is not simple, but some optimum values should be found out. The aim of the residual matrices is to remove the low frequency regional trends from a set of data, leaving, presumably with a minimum of distortion, the higher frequency variations due to local geologic trends (Fuller 1967).

A computer program was devised to calculate the nrrms between the theoretical and the filtered regional fields for the different combinations of M and Q values of various depth units (Z) (Fig. 1), where M is the number of coefficients of the side of the square matrix C, and Q is the regionalization parameter, to help achieve the previously stated goal. Figure 1 demonstrates that the goodness of fit (lower values of nrrms) is improved as M and Q increase. However, the rate of this improvement is not constant. It is particularly constant and insignificant beyond certain values of M and Q (21 and 60). For less values of M and Q the goodness of fit (and hence the efficiency of the low pass filter) declines rapidly. In the region where the goodness of fit between the theoretical and filtered fields improves, the set of coefficients (weights) is obviously too large in areal extent. To be of practical value, the matrix should be shortened. It is clear from Fig. 1 that a choice of certain low pass filter parameter Q and certain matrix length M is crucial to the accuracy of the resulting separation of the regional field. It is seen that the output curves of nrrms beyong $M = 21$ and $Q = 60$ acceptably lie very close to each other, and could apply to get these chosen optimum parameters.

A family of filtered regional profiles is reproduced on Figs. 2, 3 and 4. Each figure represents the filtered and theoretical regional fields for different values of M at the chosen $Q = 60$, and for different Q values at the chosen $M = 21$, at the three assigned depth units ($Z = 1,2$ and 3). It is clear from these three figures that faint secondary side anomalies (noise) occur at places where in reality there are none. These

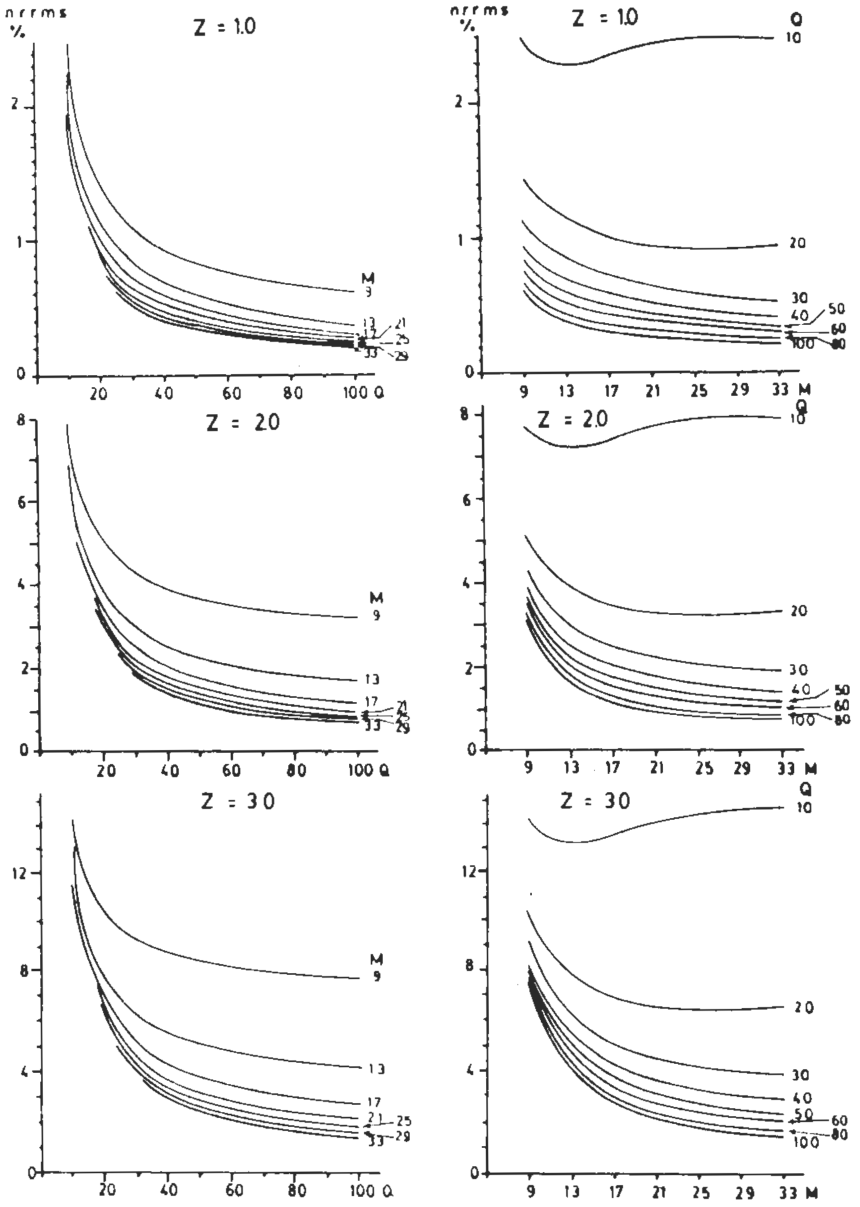

FIG. 1. Change of the efficiency of filtering (normalized residual root mean square "nrrms" in percent) with different lengths of the filter matrices (M) and low pass filter parameters (Q), at various depth units (Z).

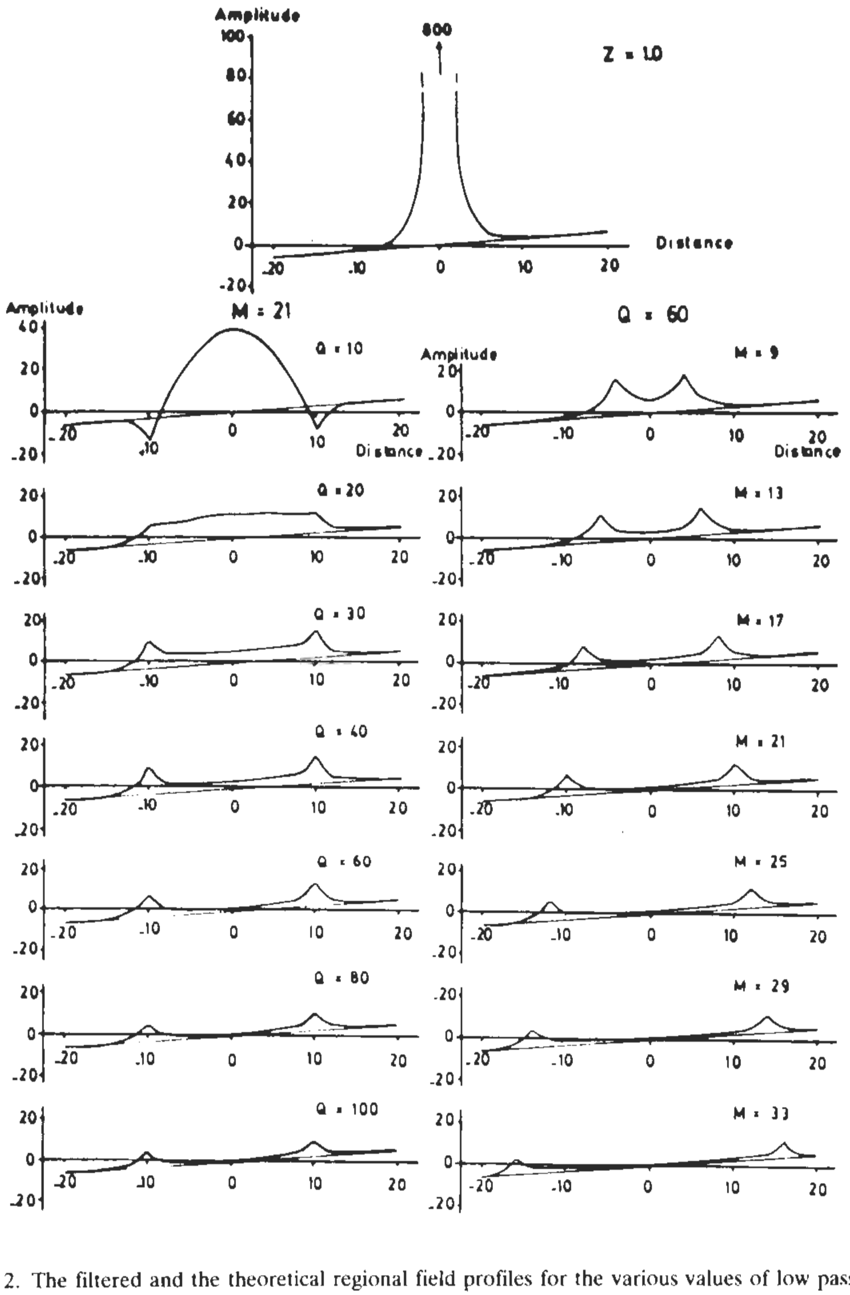

FIG. 2. The filtered and the theoretical regional field profiles for the various values of low pass filter parameters (Q) at the chosen length of the filter matrix (M = 21) and for various M values at the chosen Q value (60), at one depth unit (Z = 1).

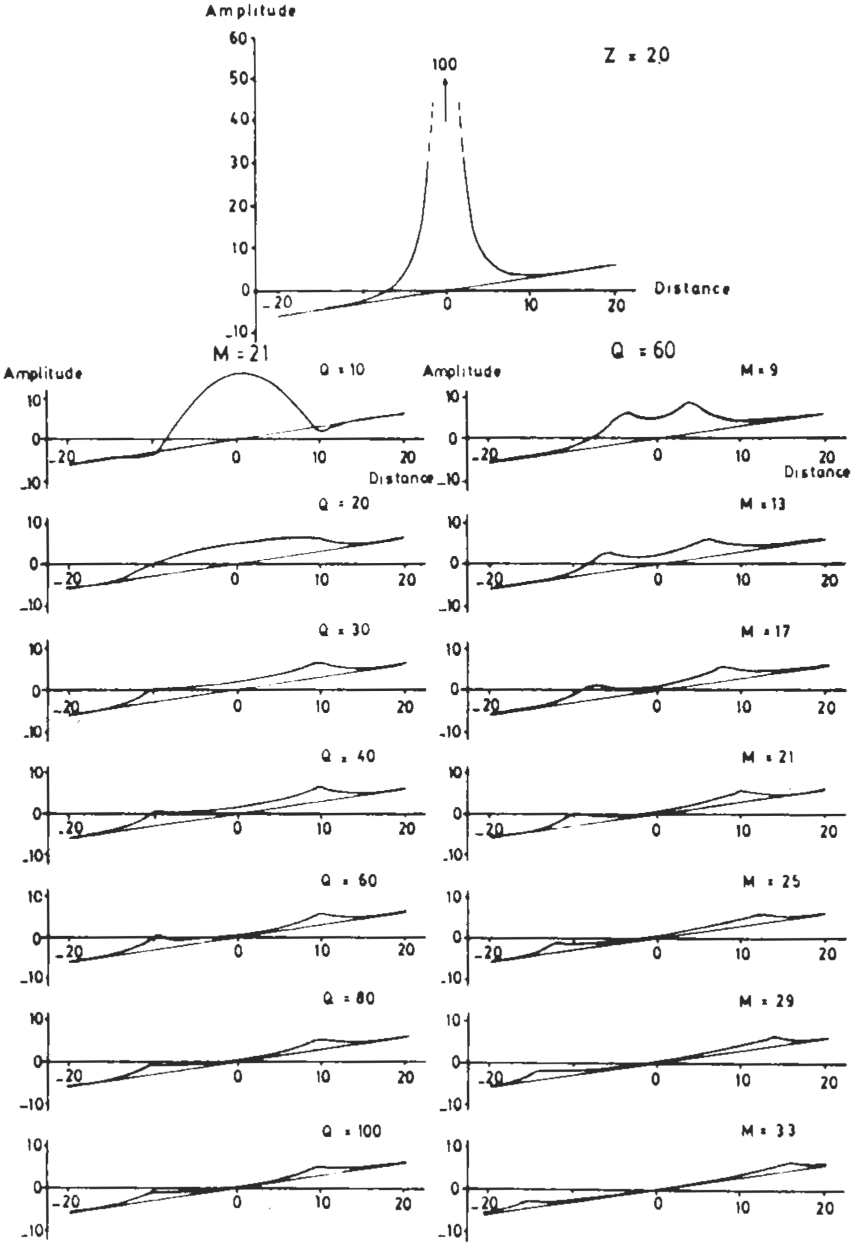

FIG. 3. The filtered and the theoretical regional field profiles for the various values of low pass filter parameters (Q) at the chosen length of the filter matrix (M = 21) and for various M values at the chosen Q value (60), at two depth units (Z = 2).

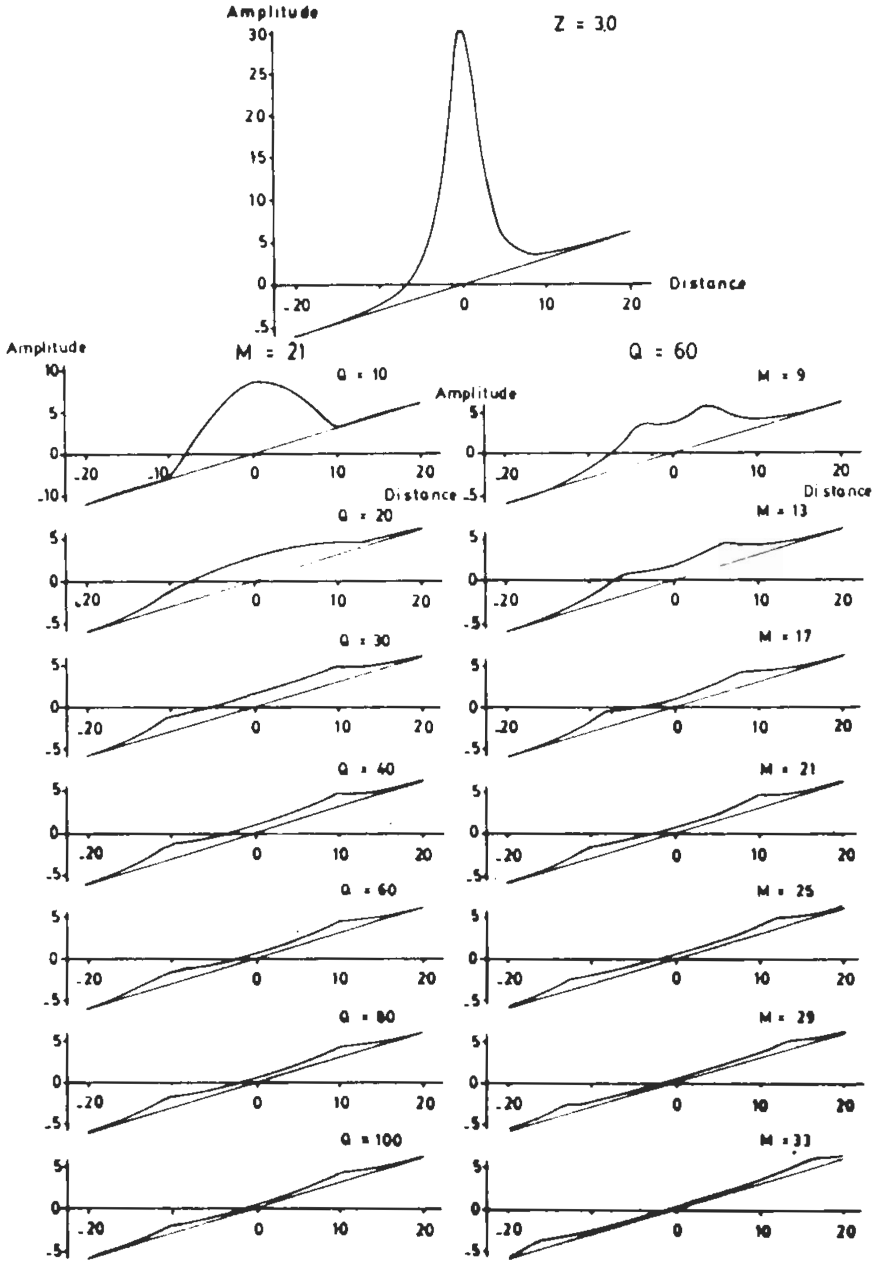

FIG. 4. The filtered and the theoretical regional field profiles for the various values of low pass filter parameters (Q) at the chosen length of the filter matrix (M = 21) and for various M values at the chosen Q value (60), at three depth units (Z = 3).

maximum differences represent those fake anomalies which appear along the output profiles at a distance equals to (M + 1)/2 from the center. At a fixed M value, the location of the noise (maximum difference) does not change with increasing Q, while its amplitude becomes weaker. On the other hand, at a fixed Q value, the location recesses away from the center and its amplitude decreases, as the value of M increases. This observation is almost the same whatever depth units may be used as shown in Fig. 5. The combination of M = 21 and Q = 60 is most probably the best, as it makes not only the position of the distortion (fake anomaly) far from the center of the anomaly, but also makes its amplitude relatively very weak.

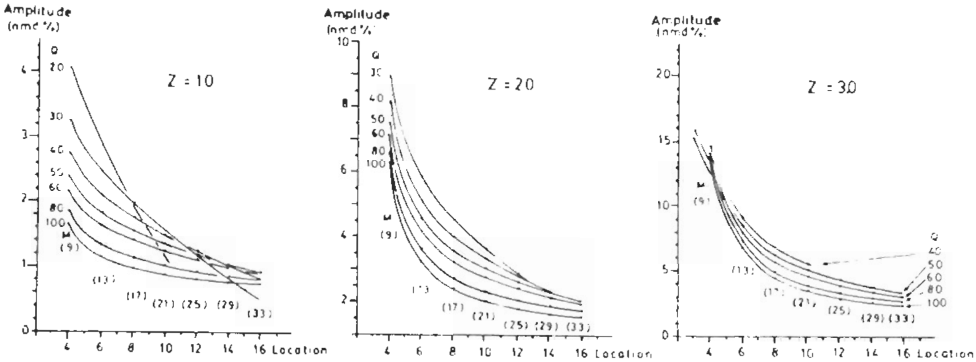

FIG. 5. Change of the location and amplitude of the normalized maximum differences (nmd in present) with the variation of the length of the filter matrix (M) and the low pass filter parameter (Q), at various depth units (Z).

Changing the value of Z has little influence on the general properties and the quality of the filter. The maximum differences (md) between the filtered and the theoretical regional or local fields (which were found to possess the same amplitude) were normalized (nmd) by relating them to their corresponding maximum amplitude of the theoretical local sphere anomaly (which depends only on the assigned depth). The behaviour of these normalized maximum differences (nmd) with the various lengths of the filter matrix (M) and with the different values of the smoothing parameter (Q) were illustrated in Fig. 6. From these plots, it could be realized that the nmd values in per cent, at the chosen M and Q values, vary between 1.25, 2.75 and 4.75 at depth units Z = 1, 2 and 3, respectively.

Similarly, the relative maximum values (rmv) for the filtered local peak to the theoretical ones in per cent were also plotted against different M and Q values (Fig. 7). The plotted curves could indicate that the filtering efficiency is very high at the chosen M and Q values (21 and 60), since these relative maximum values in per cent are 99.794, 99.068 and 97.691, at Z = 1,2 and 3 depth units, respectively (Table 1).

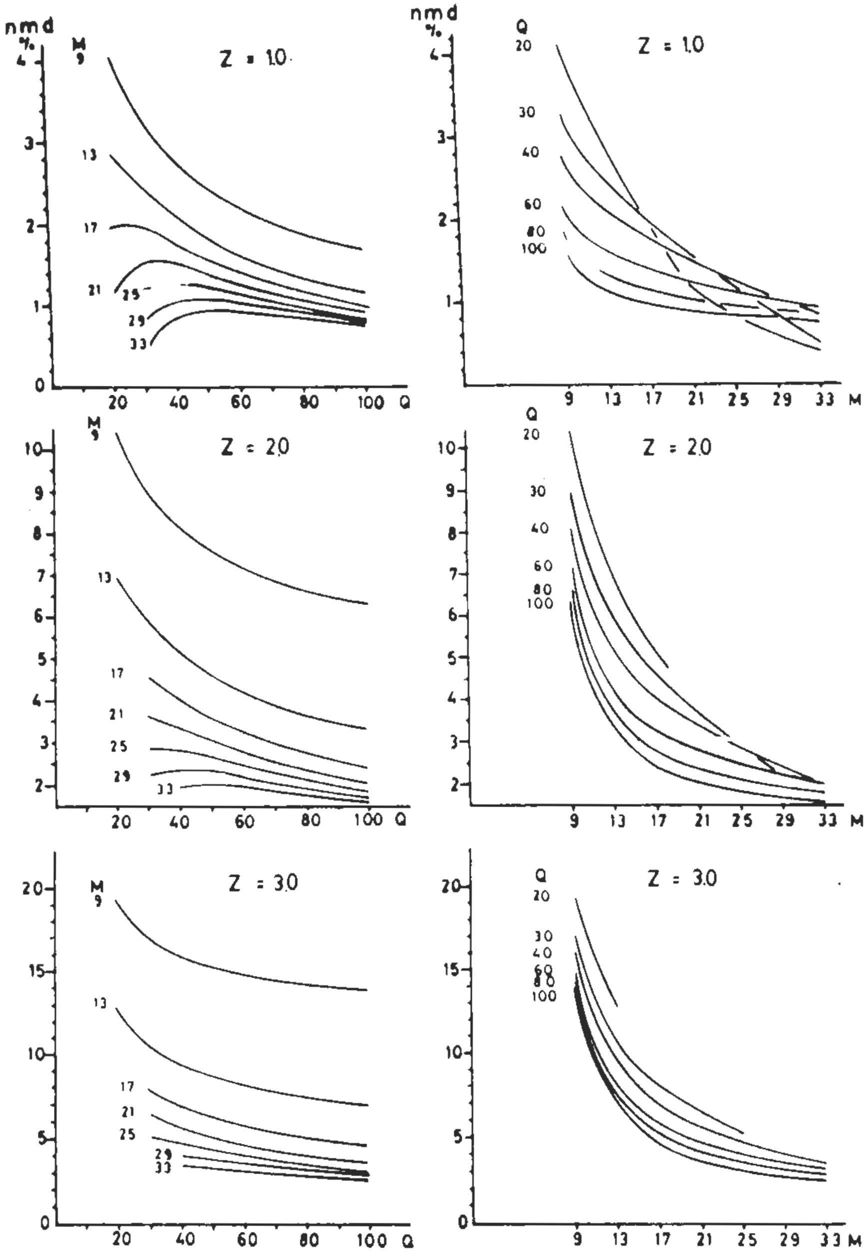

FIG. 6. Change of the amplitude of the normalized maximum differences (nmd in percent), between the theoretical and the filtered fields, with variation of the length of the filter matrix (M) and the low pass filter parameter (Q) at various depth units (Z).

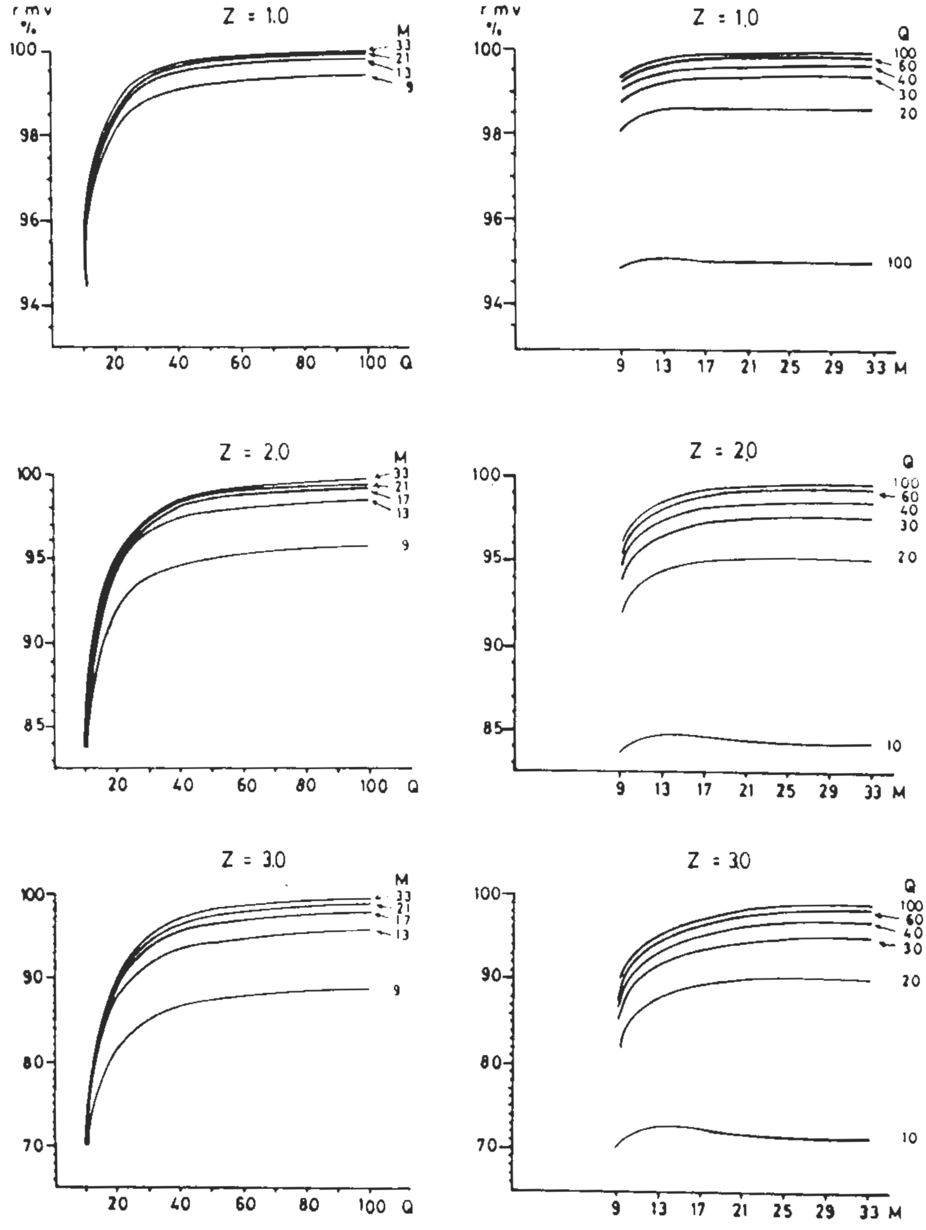

FIG. 7. Change of the relative maximum values (rmv in percent) of the filtered and the theoretical fields with variation of the length of the filter matrix (M) and the low pass filter parameter (Q) at various depth units (Z).

TABLE 1. Summary of the results of the study including the location and amplitude of the nmd %, nrrms %, and rmv % at the three depth units, for different Q values at the chosen M = 21, and for different values of M at the chosen Q = 60.

M^{α}	Q^{c}	Location	nmd** Amplitude %			nrrms %***			rmv %****		
			Z* = 1	Z = 2	Z = 3	Z = 1	Z = 2	Z = 3	Z = 1	Z = 2	Z = 3
21	10	±10				2.440	7.672	13.976	95.003	84.318	71.822
	20	10				0.937	3.266	6.531	98.597	95.072	90.162
	30	10	1.547	3.607		0.648	2.223	4.553	99.333	97.485	94.630
	40	10	1.495	3.352	5.640	0.529	1.774	3.675	99.599	98.388	96.355
	50	10	1.368	3.035	5.110	0.454	1.509	3.116	99.724	98.824	97.205
	60	10	1.241	2.751	4.668	0.399	1.329	2.821	99.794	99.068	97.691
	80	10	1.031	2.312	4.009	0.323	1.095	2.393	99.864	99.322	98.204
	100	10	0.878	2.002	3.554	0.273	0.949	2.130	99.900	99.446	98.464
9	60	± 4	2.163	7.136	14.742	0.765	3.494	8.124	99.231	95.275	87.705
13		6	1.601	4.177	8.090	0.527	2.060	4.779	99.647	98.000	94.580
17		8	1.381	3.223	5.751	0.445	1.562	3.453	99.755	98.775	96.800
21		10	1.241	2.751	4.668	0.399	1.329	2.821	99.794	99.068	97.691
25		12	1.124	2.432	4.023	0.365	1.191	2.474	99.811	99.201	98.106
29		14	1.008	2.174	3.554	0.336	1.093	2.244	99.820	99.268	98.319
33		16	0.902	1.944	3.166	0.311	1.015	2.072	99.824	99.304	98.437

α Matrix length (M).
c Low pass filter (regionalization) parameter (Q).
* Depth to the causative body (Z).
** Normalized maximum differences (nmd).
*** Normalized residual root mean square (nrrms).
**** Relative maximum values (rmv).

Figure 8 has been drawn to illustrate the relationship between the amplitude of the nmd in per cent and the different depth units at the chosen M and Q values (21 and 60). It could be concluded that this relation is approximately linear in this depth range, where the amplitude of the maximum difference ranges between 0.5 and 4.7% as related to the peak value of the local theoretical anomaly.

Figure 9 has been plotted to show the relation between the relative maximum values (rmv) of the filtered local field to that of the theoretical one (in per cent) at different depth units. This relationship, which is approximately linear, was calculated using the chosen filter matrix length (M = 21) and the low pass filter parameter (Q = 60). Thus, applying this representation (Fig. 9), the depth to the causative spherical bodies could be determined knowing the relative maximum values in per cent of their filtered and observed fields, and taking into consideration that the regional field is alike the assumed one.

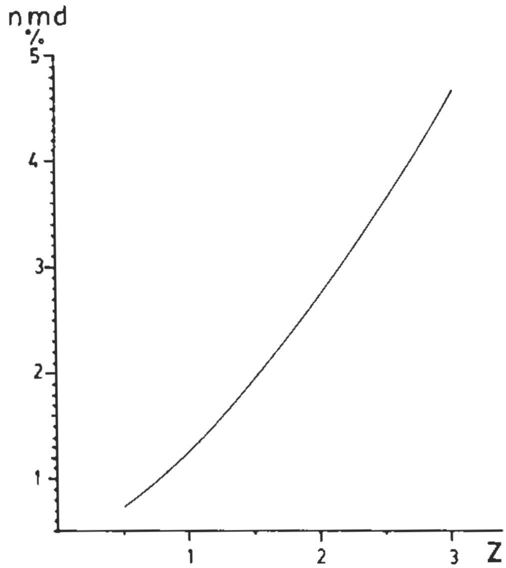

FIG. 8. Relation between the amplitude of the normalized maximum differences in percent (nmd) with various depth units (Z), at the chosen values of the length of the filter matrix (M = 21) and the low pass filter parameter (Q = 60).

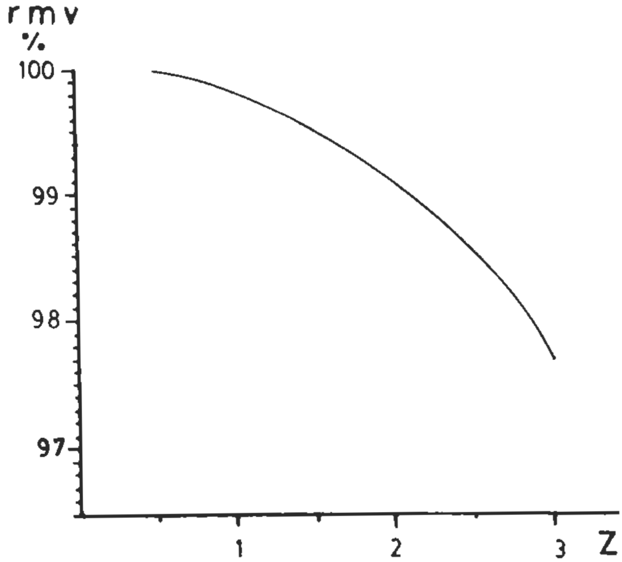

FIG. 9. Relation between the relative maximum values (rmv in percent) of the filtered and the theoretical fields with various depth units (Z), at the chosen values of the length of the filter matrix (M = 21) and the low pass filter parameter (Q = 60).

The results of the study including the location and the amplitude of the nmd in per cent, nrrms in per cent, and rmv in per cent at the three depth units, for a sequence of several different Q values at the chosen M = 21, and for another sequence of several different values of M at the chosen Q = 60, are summarized in Table 1.

Conclusions

The present work demonstrates the effect of the choice of the low pass filter parameter (Q) and the matrix length (M) on the efficiency of the preparation of the regional component of the potential field data. It also presents an evaluation of the efficiency of filtering by means of a measure of the goodness of fit between the theoretical and the filtered data. The study makes it clear that the filter parameter and the matrix length suitable for the separation of the effect of deep local bodies can also be efficiently used for separating the effect of the shallower ones.

It has been found that an increase in the number of coefficients (matrix size) improves the efficiency of the low pass filter starting from Q = 40 at various depth units.

It could be observed that the output curves of the normalized rrms in per cent beyond a certain low pass filter parameter (Q = 40) and a certain matrix length (M = 17) are very close to each other.

It was also found that fake anomalies (maximum differences) appear along the output profiles (filtered ones) at distances equal to (M + 1)/2 from the centre.

The result of the study of the effect of variation of M and Q on the resultant filtered field, as concerning either the fake side anomaly (maximum difference) or the central peak value, revealed that the choice of M = 21 and Q = 60 for different depths is a reasonable compromise between the practical matrix size and the required efficiency of the lower pass filter.

The plotted relative maximum values of the filtered field to that of the original (theoretical) field versus the different depth units could be of significance in the prediction of depth to the causative spherical bodies in case of regional fields similar to the assumed one (*i.e.*, planar regionals).

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تقويم كفاءة مرشحات المرور الخفيض المُقصَّرة المحسوبة بوساطة تحويل فورييرالعكسي لمجالات الجهد للأجسام الكروية والإقليميات المستوية

أحمد أبو بكر باعمار(١) ، قدري محمد فؤاد(٢) ومجدي لوقا مليك(٢)
كلية علوم الأرض – جامعة الملك عبد العزيز(١) – جدة – المملكة العربية السعودية
وهيئة المواد النووية – طريق المعادي القطامية – القاهرة – مصر(٢)

يمكن تحديد القصد من مرشح الفصل ، بأنه إخماد الشذوذات ذات المصدر العميق وإبراز الشذوذات ذات المصدر الضحل . مثل هذا الفصل يمكن إنجازه عن طريق تحويل المعطيات الشبكية المتسامتة باستخدام مجموعة من المعاملات المحسوبة بالتقويم العددي لتحويل فورييرالعكسي لمصفوفة المرشح .

لقد تم انتقاء المجال المتبقي لبنية كروية اختيارية لاختيار وتقويم هذه الطريقة ، وذلك بهدف الحصول على مصفوفة مختصرة ، باستخدام النموذج الكروي والاقليمي المستوى لتبسيط الحسابات فقط . وقد حُسبت كفاءة مرشح المرور الخفيض إحصائيا عن طريق قياس الجذر المتوسط التربيعي المتبقي السوي بين كل من المجالين النظري والمرشح .

تمت دراسة كفاءة الترشيح الناتجة عن استخدام معاملات وأحجام مصفوفات لمرشحات المرور الخفيض . كما تم رسم منحنيات علاقات متبادلة متنوعة فيما يختص بالمعامل الإقليمي ، وطول المصفوفة ، وعمق الجسم المسبب لشاذة مجال الجهد ، وذلك لاختيار مصفوفة تقارب ماأمكن استجابة المرشح النظرية . وقد تبين أن اختيار مرشح للمرور الخفيض ذا معامل إقليمي (تنعيمي) قدره ٦٠ في مصفوفة طولها ٢١ يمثل مواءمة بين حجم المصفوفة العملي والكفاءة المتطلبة في مرشح المرور الخفيض عند وحدات عمق متفاوتة . ويستخلص من ذلك أن معاملات المرشح وأطوال المصفوفة التي تلائم فصل الأجسام الكروية المحلية العميقة ، يمكن استخدامها كذلك بكفاءة للأجسام الضحلة .